

Impulse Response Functions (IRFs)

Macroeconomics (M8674)

February 2026

Vivaldo Mendes, ISCTE

vivaldo.mendes@iscte-iul.pt

1. Main goal

What are IRFs?

- Impulse response functions represent the response of the endogenous variables of a given system, when one (or more than one) of these variables is hit by an exogenous shock.
- The *nature* of the shock can be:
 - Temporary
 - Permanent
 - Systematic
- *Linear systems*. The magnitude of the shock does not change the stability properties of the system.
- *Nonlinear systems*. In this case, the magnitude of the shock is of great importance and it can change the stability of the system under consideration.

An example

- Consider the simplest case, an **AR(1)**:

$$y_{t+1} = ay_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, 1) \quad (2)$$

- Assume that for $t \in (1, n)$:

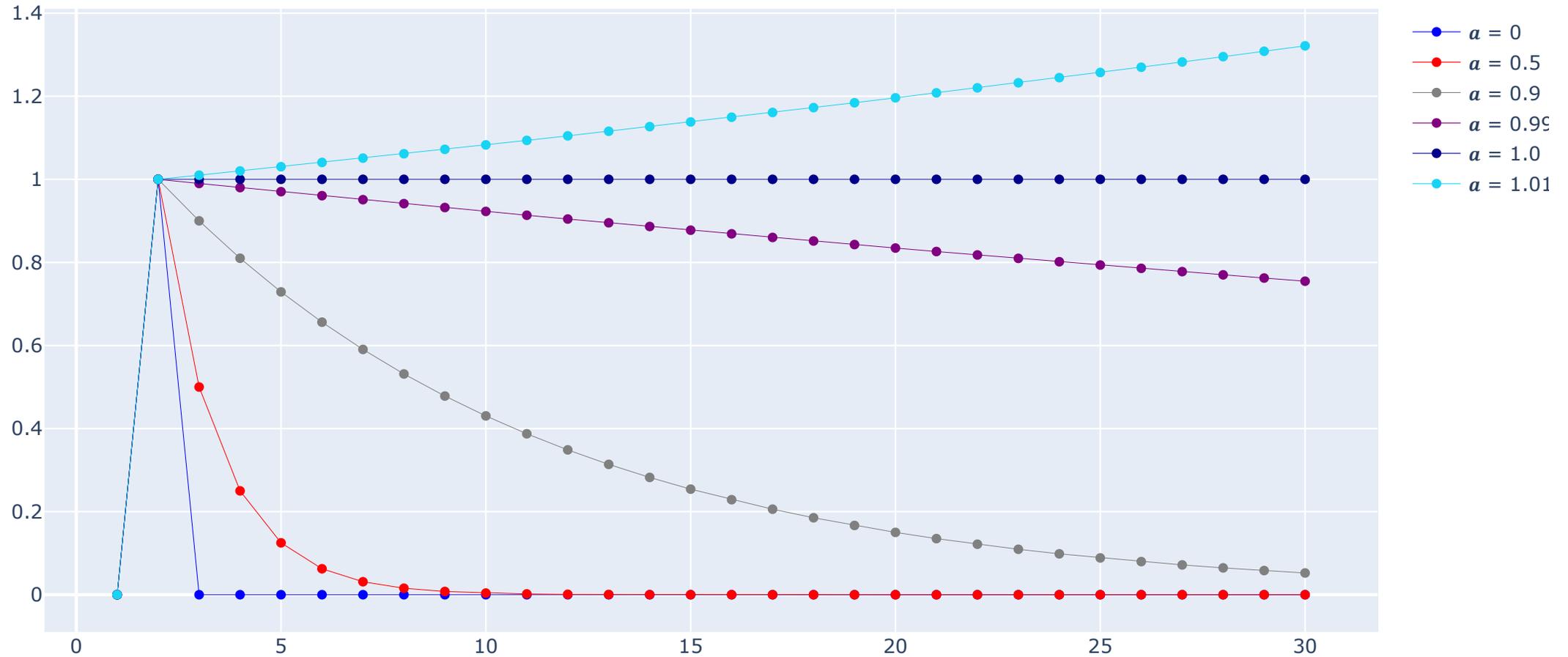
$$y_1 = 0 ; \varepsilon_2 = 1 ; \varepsilon_t = 0 , \forall t \neq 2$$

- This implies that at $t = 2 \Rightarrow y_2 = 1$. But what happens next, if there are no more shocks?
- The IRF of y provides the answer.
- The dynamics of y will depend crucially on the value of a . **Six examples:**

$$a = \{0, 0.5, 0.9, 0.99, 1, 1.01\}$$

The IRFs of the AR(1) Process

Impulse Response Functions (IRF) from an AR(1) process



Another example

- Consider a more sophisticated case, an **AR(2)**:

$$y_{t+1} = ay_t + by_{t-1} + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, 1)$$

- Assume that for $t \in (1, n)$:

$$y_1 = 0 ; y_2 = 0 ; \varepsilon_3 = 1 ; \varepsilon_t = 0 , \forall t \neq 3$$

- This implies that at

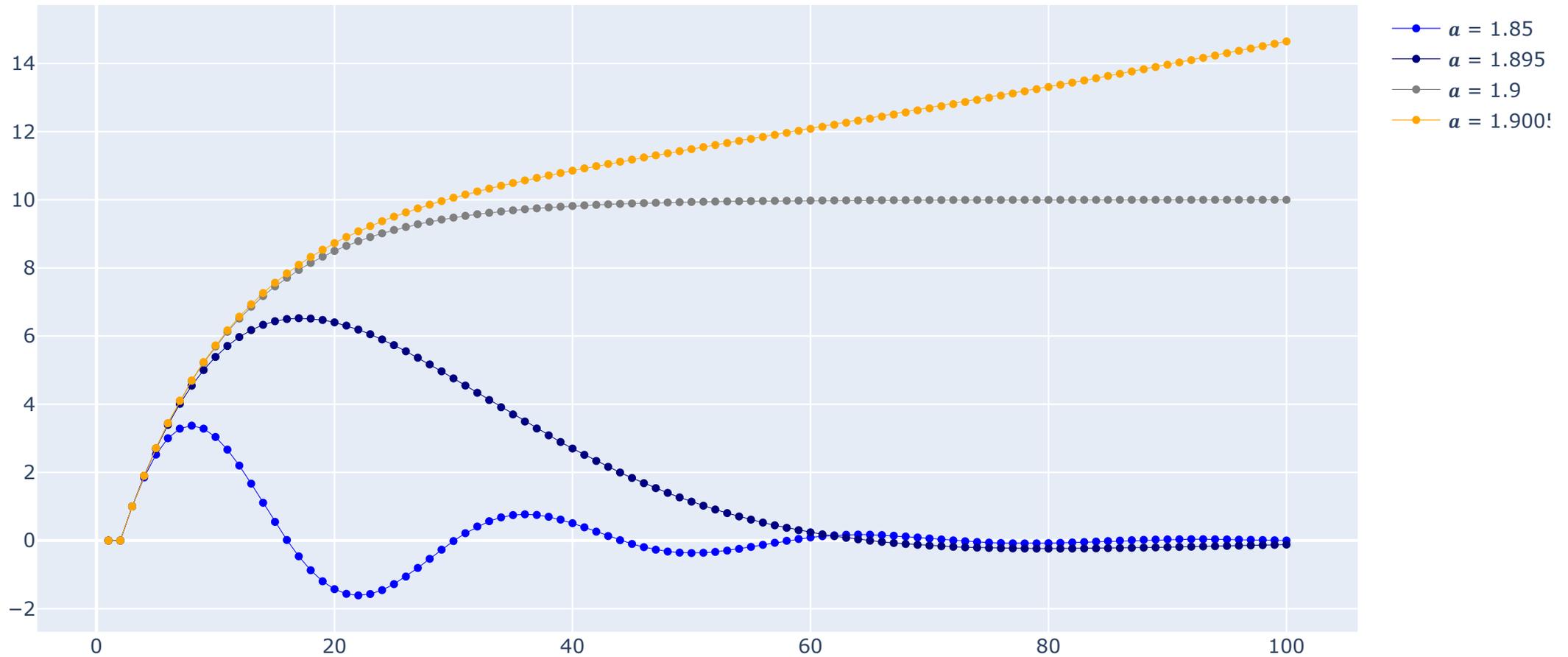
$$t = 3 , \varepsilon_3 = 1 \Rightarrow y_3 = 1.$$

- What happens next, if there are no more shocks? The IRF of y provides the answer.
- The dynamics of y will depend on the values of a and b . For simplicity consider:

$$b = -0.9 ; a = \{1.85, 1.895, 1.9, 1.9005\}$$

The IRFs of the AR(2) Process

Impulse Response Functions (IRF) from an AR(2) process



More Sophisticated Examples

- A similar reasoning can be applied to our rather more general model:

$$X_{t+1} = A + BX_t + C\varepsilon_{t+1} \quad (3)$$

- .. where B, C are $n \times n$ matrices, while $X_{t+1}, X_t, A, \varepsilon_{t+1}$ are $n \times 1$ vectors.
- Consider the following **VAR(3)** model:

$$X_{t+1} = \begin{bmatrix} z_{t+1} \\ w_{t+1} \\ v_{t+1} \end{bmatrix}$$

- In this example we take matrices A , B and C given by:

$$A = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.97 & 0.10 & -0.05 \\ -0.3 & 0.8 & 0.05 \\ 0.01 & -0.04 & 0.96 \end{bmatrix}, \quad C = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}.$$

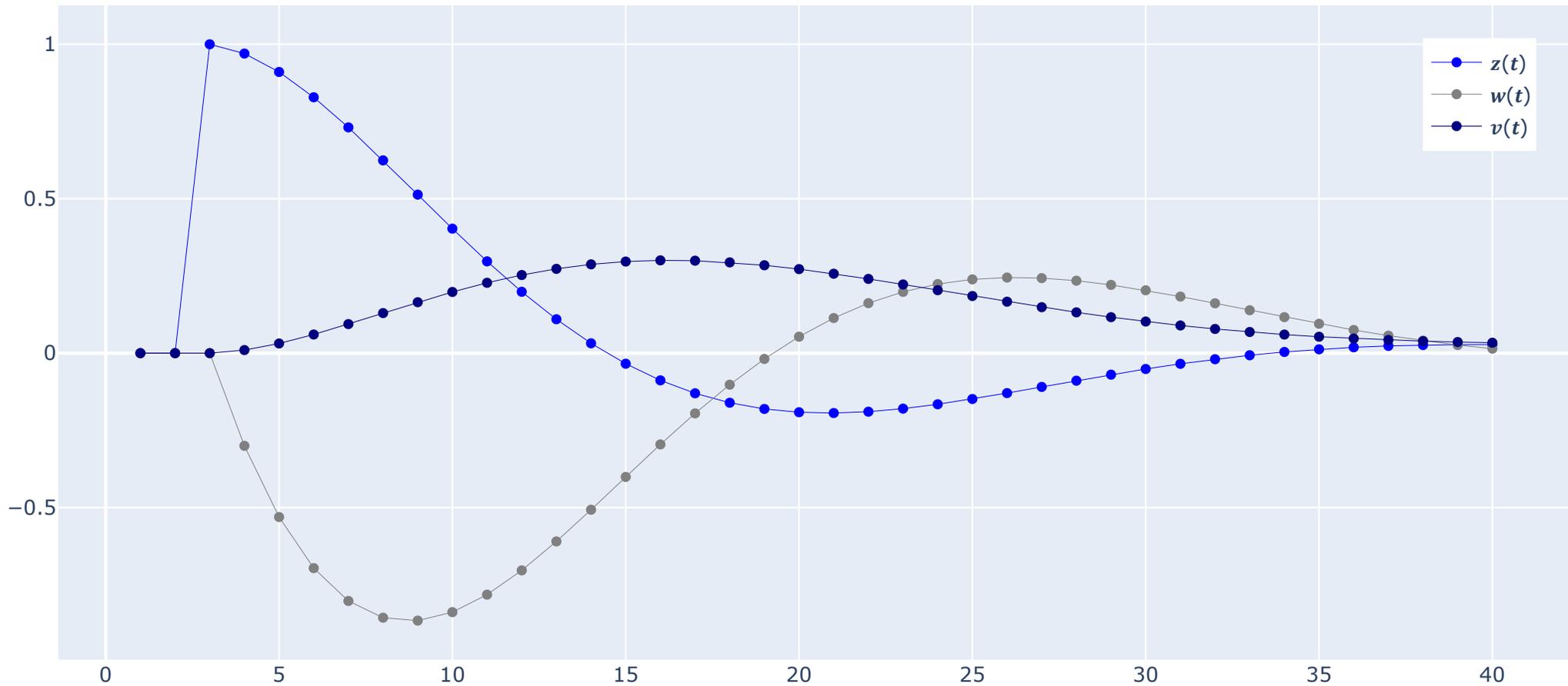
- The initial state of our system (or its initial conditions) are: $z_1 = 0$, $w_1 = 0$ and $v_1 = 0$, that is:

$$X_1 = [0, 0, 0]$$

- The shock only hits the variable z_t (notice the blue entry in matrix C), and we assume that the shock occurs in period $t = 3$.
- What happens to the dynamics of the three endogenous variables? See next figure.

The IRFs of our VAR(3) Process

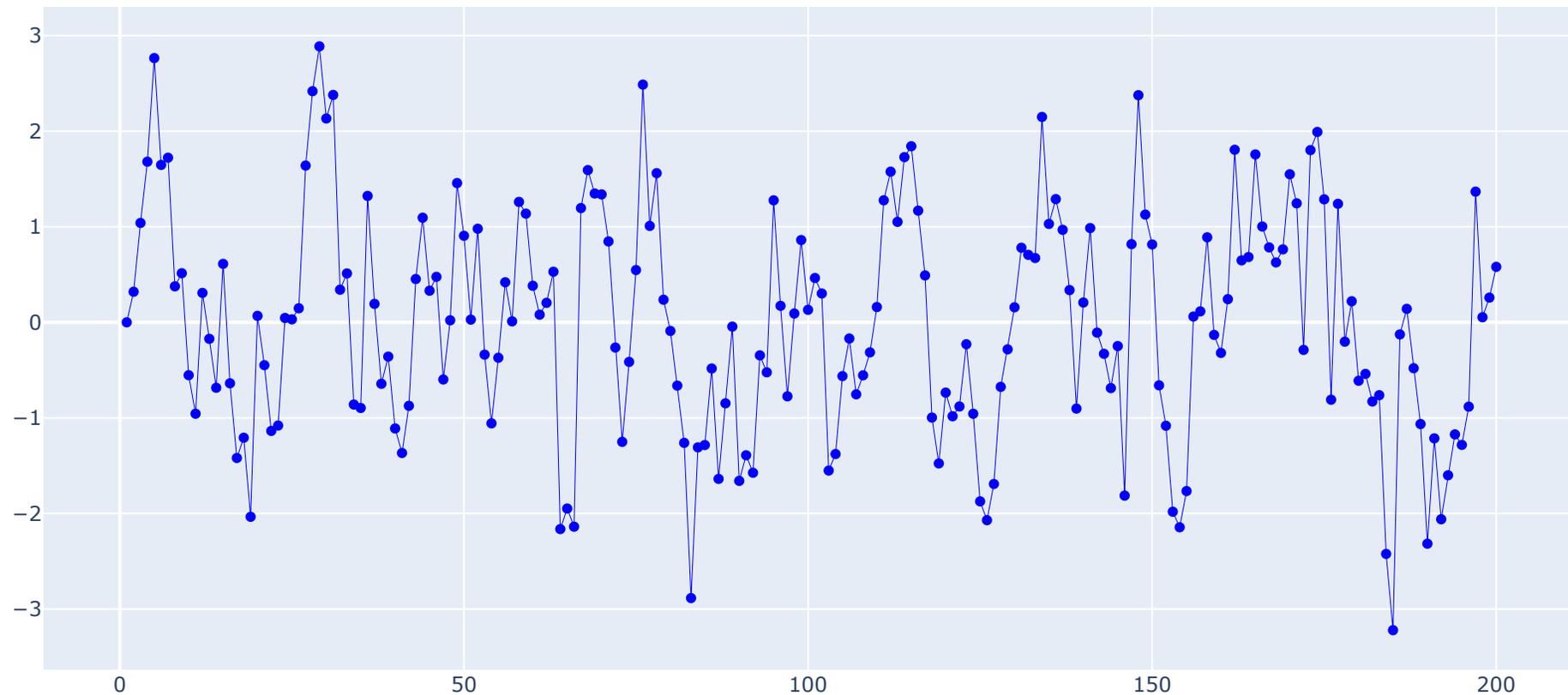
Impulse Response Functions (IRF) from a VAR(3) process



AR(1): A Sequence of Shocks

Consider the same AR(1) as in eq. (2). But now impose a *sequence of 200 shocks*.

A stochastic AR(1) process with mean=0 and a sequence of shocks



Implications of a Linear Structure

- In the previous examples, the structure of all our models *was linear*.
- This has a crucial implication:
 - *The shock's magnitude did not alter the dynamics produced by the shock itself.*
 - *Only the structure of the model would lead to different outcomes.*
- This does not usually occur if the structure of the model is *non-linear*. In this case, the magnitude of the shock may produce different outcomes even if the system's structure remains the same.
- We do not have time to cover this particular point.
- But be careful: *if the structure of the model is non-linear, large shocks can not be simulated ... in a linearized version of the original system.*

3. Macroeconomics: Why are IRFs important ?

Macroeconomics: real experiments are impossible

- In physics, chemistry, biology, etc., we *can perform experiments* to study the behavior of a system when it is hit by a shock.
- In macroeconomics, we *cannot perform such experiments*.
- So, we construct abstractions -- *models* -- to simulate how the economy is supposed to work
- We assume that the economy is on its *steady-state* and we impose a certain shock to an endogenous variable.
- The IRFs give us the *reaction of the entire economy*. So, in a sense, IRFs represent our "experiments".

IRFs everywhere

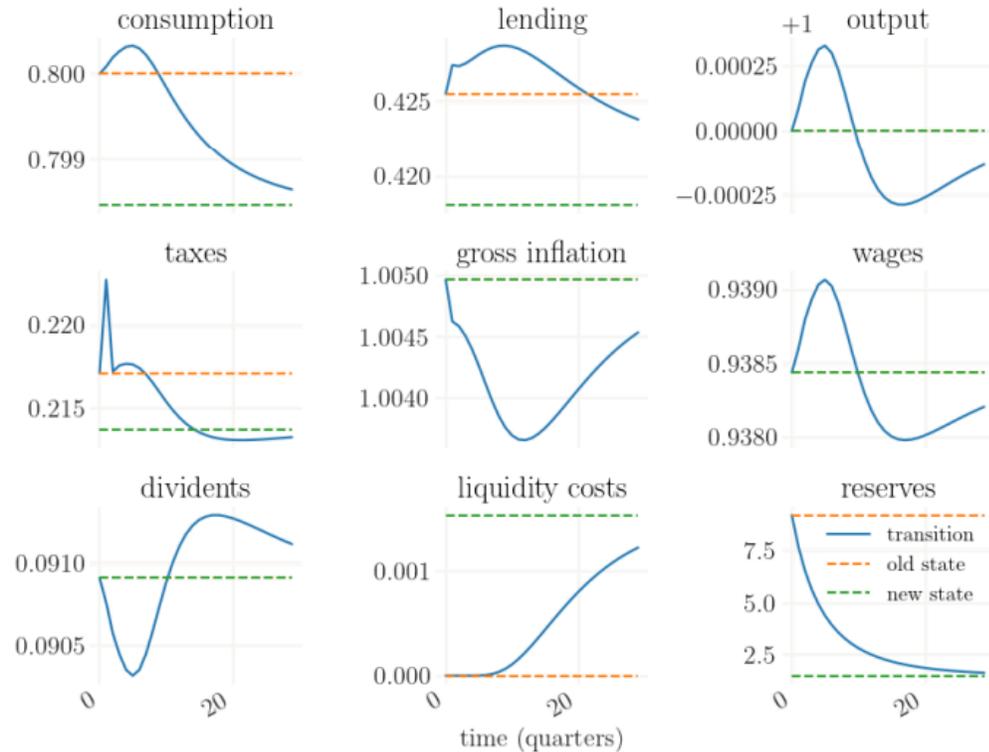


Figure B.11: Aggregate dynamics for the transition from 20 times the MRR to the welfare optimum from Figure 5 at $\frac{J}{\psi D} = 2.833$.

A good example:

- Boehl, Gregor (2025). The Political Economy of Monetary Financing without Inflation, Working Paper, University of Bonn.
 - An initial steady state
 - A new steady state
 - IRFs as the transition process

3. Readings

- For this point, there is no compulsory reading. However, any modern textbook on time series will cover this subject.
- At an introductory level, see sections 11.8 and 11.9 of the textbook: Diebold, F. X. (1998). *Elements of forecasting*. South-Western College Pub, Cincinnati.
- At a more advanced level, see, e.g., section 2.3.2 of the textbook: Lütkepohl, H. (2007). *New introduction to multiple time series analysis* (2nd ed.), Springer, Berlin.